

# On the Physical Basis of Power Losses in Laminated Steel and Minimum-Effort Modeling in an Industrial Design Environment

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**Abstract** – A procedure is described for identifying a general mathematical model of core losses in ferromagnetic steel based on a minimal amount of experimental data. This model has a hysteresis loss multiplicative coefficient variable with frequency and induction and a combined coefficient for eddy-current and excess losses that is also variable with frequency and induction. A physical interpretation and a test procedure for the identification of the core loss coefficients are proposed. Validation was successfully performed on a number of different samples of non-grain oriented fully and semi-processed steel alloys.

**Index terms** – core loss, eddy-current loss, hysteresis loss, Epstein test, ferromagnetic steel

## I. INTRODUCTION

For the last couple of decades, the model of core losses, originally developed by Bertotti served as a main reference for the electrical machines engineering community [1-2]. In order to account for “excess” or “anomalous” losses that could not be explained through the conventional power loss theory, a supplementary term was consecrated. The additional term – which was defined as a function of frequency and induction, each at a power of 1.5 – also contributed to an improvement of the fit between a material model with constant coefficients and experimental data, when a relatively limited range of frequency and induction is considered.

More recent research studies, performed on an extensive collection of samples of fully and semi-processed cold rolled laminated steel, demonstrated that a better fit, over a substantially wider range of frequencies between 20 and 2 kHz and inductions from 0.05 up to 2T, can be achieved by a model which contains only two terms – one for hysteresis losses and one for the combined effect of eddy currents and anomalous losses – but has variable coefficients [3]. Our previous research was mainly based on original best-fit methods [3, 4], a practical engineering approach which was also favoured by other authors, e.g. [7, 8]. The present paper describes new efforts aimed in particular at explaining the physical nature of the core loss coefficient variation with frequency and induction, and also at the identification of general trends that can be conveniently exploited to minimize the experimental and numerical effort required for material modeling.

Following a brief review of the model employed, the loss production mechanism is discussed with reference to the low frequency hysteresis cycles, leading to the

identification of the hysteresis loss coefficient and its dependency of induction. For the combined effect of eddy current and anomalous losses, explanations and hypotheses related to the Brakhausen effect and the instability of the magnetic domains are provided in order to support the variability of the associated loss coefficient both with frequency and induction. Another section is devoted to a proposed algorithm that involves a minimum of four measurements at different inductions and a given low frequency and another four measurements of variable induction at a constant frequency, conveniently selected approximately in the middle of a relatively wide range of frequency, for which the specific core losses are modeled. The paper is illustrated with extensive examples provided by one electric steel of the semi-processed type and one of the fully-processed type (see Appendix).

## II. CORE LOSSES MODELING IN ELECTRIC STEEL LAMINATIONS

The most employed model in the work published over the last decades is based on that developed by Bertotti [1] for laminations excited with sinusoidal magnetic flux, as in an Epstein test. According to this model, the specific iron losses are a summation of hysteresis, classical eddy-current and excess loss:

$$w_{Fe} = k_h f B^\alpha + k_e f^2 B^2 + k_a f^{1.5} B^{1.5} \quad (1)$$

The coefficients  $k_h$ ,  $k_e$ ,  $k_a$  and  $\alpha$  are determined from the measured data for a certain frequency by using the least square method and are assumed to be constant.

In practice the specific iron-loss for the same grade material varies in between coils and batches within acceptable manufacturing tolerances. The results obtained using the model (1) illustrate the fact that the loss coefficients  $k_h$ ,  $k_e$ ,  $k_a$  need to be dependent of both flux density and frequency and that the usual approach of constant coefficients can lead to unpredictable and significant numerical errors. To account for loss coefficients variation with induction and frequency we are analysing the model CAL2 and allows hysteresis loss coefficient to vary with frequency while the classical loss and anomalous loss are grouped into an eddy-current loss term, where the corresponding coefficient varies with induction and frequency level:

$$w_{Fe} = k_h(f, B) \cdot f B^2 + k_c(f, B) \cdot (f B)^2 \quad (2)$$

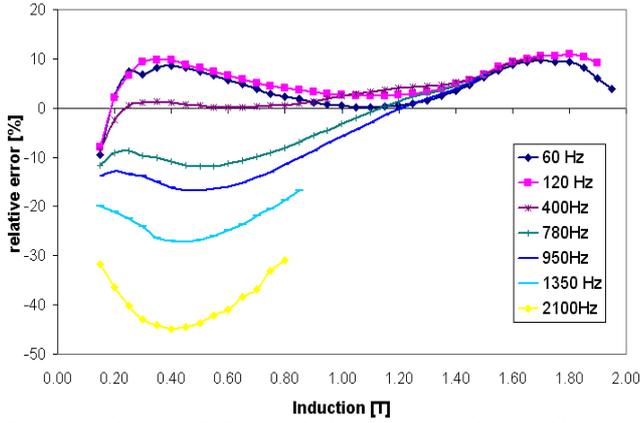


Fig. 1 Errors between the losses calculated by the model (1) with constant coefficients and Epstein measurements – fully-processed steel M43.

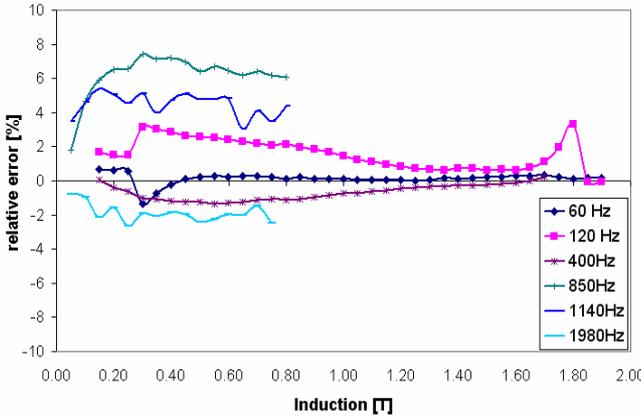


Fig. 2 Errors between the losses calculated by the model (2) with variable coefficients and Epstein measurements – fully-processed steel M43

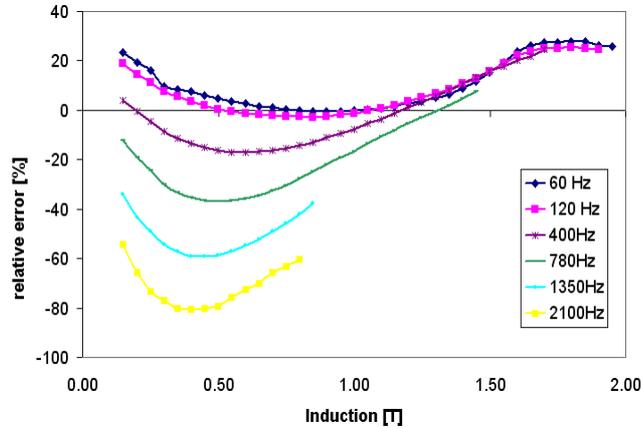


Fig. 3 Errors between the losses calculated by the model (1) with constant coefficients and Epstein measurements – semi-processed steel SP

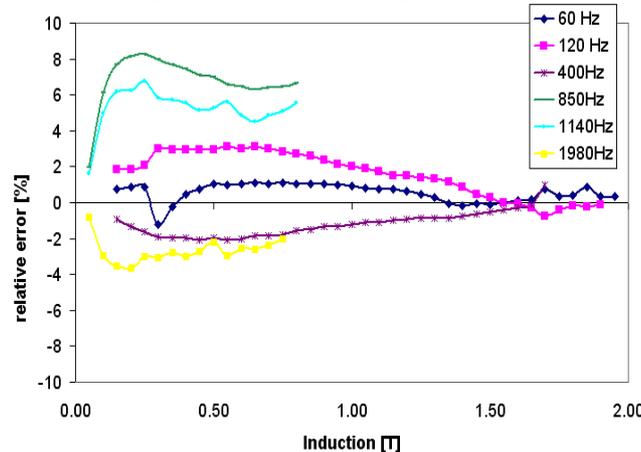


Fig. 4 Errors between the losses calculated by the model (2) with variable coefficients and Epstein measurements – semi-processed steel SP

The authors have successfully fitted the model (2) to laminated steels typically employed in electric motor manufacturing, such as semi-processed cold rolled electric steel, which after annealing have the main characteristics listed in Appendix (generically denoted as SP) and widely available fully-processed material (generically denoted as M43). Different gauges have been studied and literally hundreds of samples were measured on a multi-frequency hysteresis-graph.

Figs. 1 and 3 show the errors between test data and estimations using the model (1) with constant coefficients, for the fully-processed steel, M43 and for the semi-processed steel SP, respectively. Figs. 2 and 4 show the errors between test data and estimations using the model (2) with variable loss coefficients for the same material.

As a first step of the procedure developed in order to identify the values of the coefficients, (1) is divided by the  $f$  and  $B^2$  resulting in:

$$\frac{w_{Fe}}{f B^2} = a + b f \quad (3)$$

$$\text{with: } a = k_h, \quad b = k_e \quad (4)$$

For any induction  $B$  at which measurements were taken, the coefficients of the above polynomial in  $f$  can be calculated by linear fitting, based on a minimum of two points. During trials, it was observed that a sample five points, represented by measurements at the same induction and different frequencies, is beneficial in improving the overall stability of the numerical procedure. From (3) and (4) the eddy-current coefficient  $k_e$  and the hysteresis loss coefficient  $k_h$  are readily identifiable.

These coefficients are independent of frequency if we use or consider a certain frequency domain for the core loss data, but unlike for the conventional model, it may exhibit a significant variation with the induction, especially for the semi-processed lamination steel material. (Figs. 5–6).

Depending on the frequency range of the core loss data considered for (3), different curves for  $k_e$  and  $k_h$  as a function of the induction are obtained.

In Figs. 5–6 the upper curves correspond to test data in a frequency range up to 400Hz, while the lower curves correspond to test data in a frequency range of up to 2000Hz. These results suggest that the eddy-current loss coefficient  $k_e$  will increase with the induction level and decrease with the frequency.

In Figs. 7–8 the lower curves correspond to test data in a frequency range up to 400Hz, while the upper curves correspond to test data in a frequency range of up to 2000Hz. These results suggest that the hysteresis loss coefficient  $k_h$  will decrease with the induction level and increase with the frequency.

Several functions have been tried to describe the loss coefficients variation with the induction level. The lowest relative error values and an easiness of the implementation are provided by third order polynomials. Hence, we may employ for curve fitting of  $k_e$  a polynomial of the form:

$$k_e(B) = k_{e3}B^3 + k_{e2}B^2 + k_{e1}B + k_{e0} \quad (5)$$

Similarly, in order to identify the coefficient  $k_h$ , a third order polynomial with the induction as variable is employed:

$$k_h(B) = k_{h3}B^3 + k_{h2}B^2 + k_{h1}B + k_{h0} \quad (6)$$

In the model CAL2 (2), within a set frequency range,  $k_e$  and  $k_h$  are dependent only on  $B$ , which provides a more straightforward computation of the loss components.

### III. PHYSICAL BASIS FOR THE LOSS COEFFICIENTS VARIATION

#### A. Hysteresis loss coefficient

The hysteresis loops are the main phenomenon in the magnetic materials behavior. There is a phase lag between the input, i.e. the field strength  $H$  and the output, i.e. the induction  $B$ . The correlation between hysteresis and losses is given by the energy conservation theory where irreversible processes take place. The area of the loop described by the variation  $B(H)$  will be the energy loss per cycle. Generally, the hysteresis losses are associated with the rate-independent loss type. The hysteresis loop for a given material strongly depend on the experimental conditions, the geometry of the sample, and practically all the chemical and metallurgical processes involved in the manufacturing of the steel laminations. At microscopic scale the magnetic domains will change their arrangement from a stable position to another (where local energy minima occur) through the so-called Barkhausen effect. Domain walls move in jumps (Barkhausen jumps), which in their nature are correlated and localized, meaning that an occurrence of one jump at a particular site increases the probability of another jump occurring in the vicinity. The product of the number of Barkhausen jumps and the average energy loss due to a single jump constitute hysteresis losses.

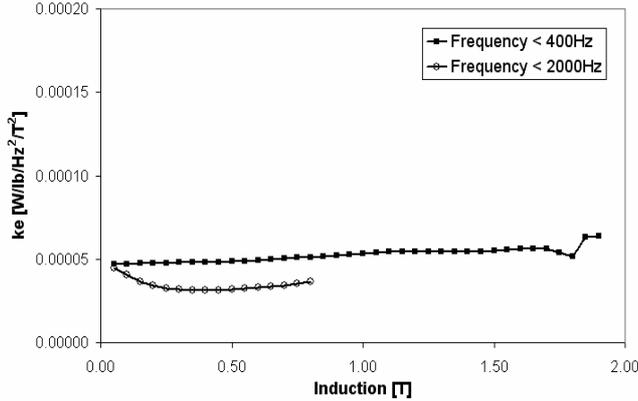


Fig. 5. Eddy-current loss coefficient  $k_e$  variation with induction within a frequency range for CAL2 model – fully-processed material M43

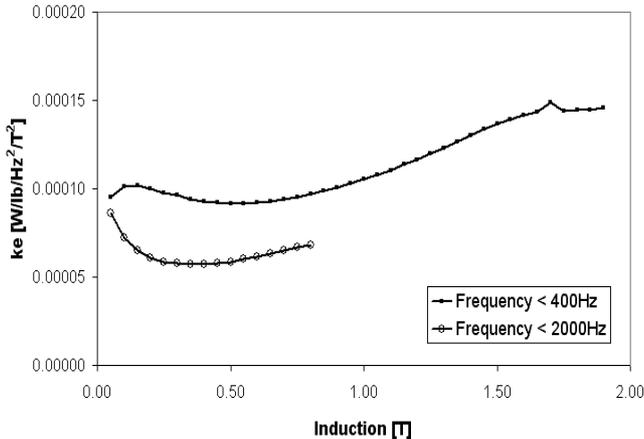


Fig. 6. Eddy-current loss coefficient  $k_e$  variation with induction within a frequency range for CAL2 model – semi-processed material SP

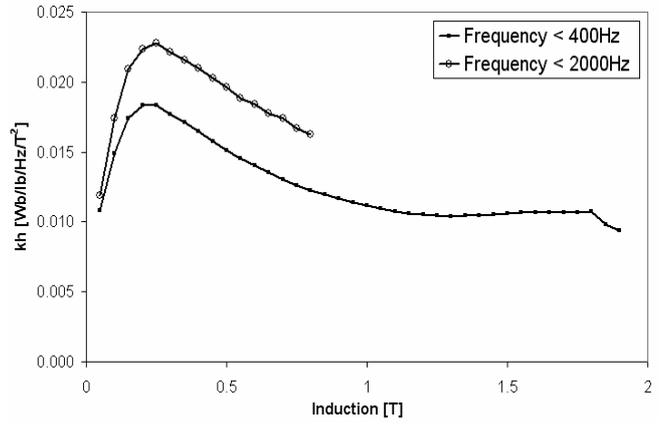


Fig. 7. Hysteresis loss coefficient  $k_h$  variation with induction within a frequency range for CAL2 model – fully-processed material M43

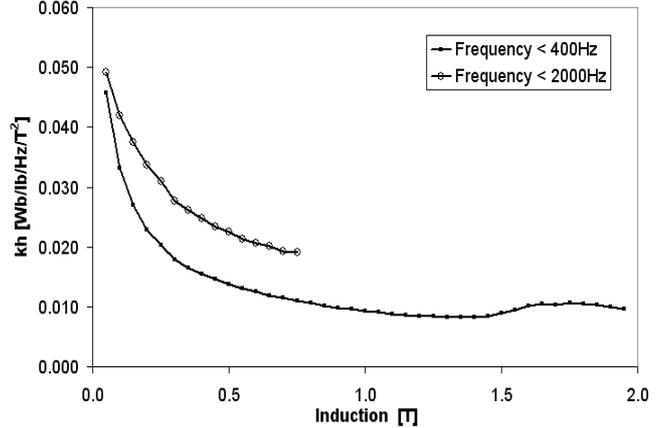


Fig. 8. Hysteresis loss coefficient  $k_h$  variation with induction within a frequency range for CAL2 model – semi-processed material SP  
 $k_h$  variation with induction.

In time-domain, the hysteresis losses may be expressed as:

$$w_h = \frac{1}{\pi T} \int_0^T \left[ k_h(f, B) \cdot B(t) \cdot \left( \frac{dB}{dt} \right) \right] dt \quad (7)$$

This equation also enables a more phenomenological explanation of the significance of  $k_h$  in the material model (2). The magnetic field  $H$  in a static hysteresis loop can be decomposed into a reversible and irreversible component and thus the specific hysteresis loss is:

$$w_h = \frac{1}{\rho_v T} \int_0^T \left[ H_{irr} \cdot \left( \frac{dB}{dt} \right) \right] dt \quad (8)$$

The irreversible field  $H_{irr}$  can be identified as the positive field value at zero induction. Based on (7) and (8),  $k_h$  can be calculated as

$$k_h = \frac{\pi}{\rho_v} \cdot \frac{H_{irr}}{B_p} \quad (9)$$

where  $B_p$  is the maximum (peak) value of induction in the hysteresis cycle and  $\rho_v$  is the volumetric mass. The coefficient  $k_h$  was estimated with this formula and data from direct hysteresisgraph measurements at peak inductions of 0.1, 0.5, 1.0, 1.5 and 1.95T and the minimum available test frequency of 5Hz (Figs. 9–10). The relative agreement with the values identified for model (2) represents additional support to the validity of the model (Figs. 11–12). Note that an alternative in determining the value of  $k_h$  is to assume

that at low frequency the losses are essentially only of hysteresis type and thus:

$$k_h = \frac{w_{Fe}}{f B^2} \quad (10)$$

The expression (9) indicates a simple method of computing the value of the hysteresis losses from the test data at low frequency, i.e. 5Hz. If it is of interest only the hysteresis loss component at a certain flux-density level, it is necessary to measure the hysteresis loop at that peak induction value and extract the value of  $H_{irr}$ , the irreversible value of the field strength. One should note that the ratio the irreversible value of the field strength over maximum value of the induction ( $H_{irr} / B_p$ ) is initially decreasing when  $B_p$  increases. Beyond the induction saturation level, i.e. generally 1.5T for standard laminated steel materials, the hysteresis loss coefficient may be practically considered constant. If we need to have an estimation of the hysteresis losses for a wide range of induction levels, it is sufficient to measure the hysteresis loops in four points, such that by employing (4) and (6) the general trend of the hysteresis loss coefficient may be extrapolated.

#### *k<sub>h</sub> variation with frequency*

The shape of the curves in Figs. 11 – 12 suggests that the explanation of the hysteresis loss coefficient with the induction level: relative permeability is variable within the saturation process of a classical single-valued  $BH$  magnetization curve. Similarly to the relative permeability variation pattern, the hysteresis loss coefficient is practically constant beyond the saturation knee point, which is around 1.5T .. 1.7T. At higher frequencies, the Barkhausen effect is more important and the magnetic domains are subjected to a higher rate of jumps from a stable local position to another. The relative permeability will have lower value for the linear region of the magnetization curves, consistent to the variation of  $k_h$  for a wider range of frequencies (See Figs. 7 – 8, high frequency polyfit curve).

#### *B. Eddy-currents loss coefficient*

The rate-dependent effects in any magnetic material determine the eddy-current losses. When measuring the dynamic hysteresis loops for the same peak induction level but for various frequency levels, we observe an increase in the loop area (Figs. 9–10). Generally, for low to medium frequencies it is assumed that the eddy-currents space-time distribution is constant within the material volume and thus the eddy-current losses are determined as resistive losses that depends on the material conductivity and lamination properties: thickness, density. In time-domain, the eddy-current losses may be expressed as:

$$w_e = \frac{1}{2\pi^2 T} \int_0^T \left[ k_e(f, B) \cdot \left( \frac{dB}{dt} \right)^2 \right] dt \quad (11)$$

The classical eddy-current coefficient is computed with the formula:

$$k_{e\_classic} = \frac{\pi^2 \sigma d^2}{6\rho_v} \quad (12)$$

where  $\sigma$  is the material conductivity,  $d$  is the lamination thickness and  $\rho_v$  is the volumetric mass density.

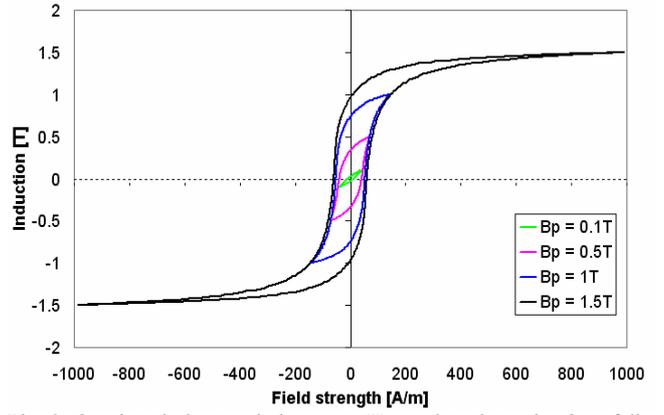


Fig. 9. Quasi-static hysteresis loops at 5Hz used to determine  $k_h$  – fully-processed material M43

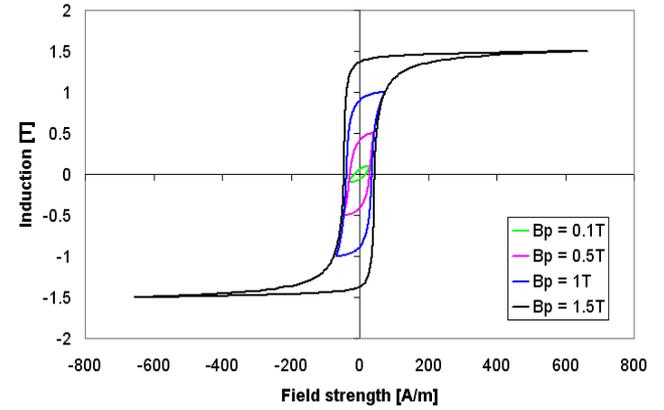


Fig. 10. Quasi-static hysteresis loops at 5Hz used to determine  $k_h$  – semi-processed material SP

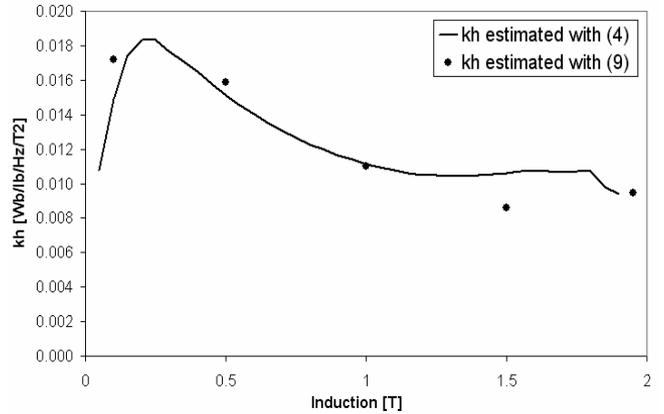


Fig. 11 Hysteresis loss coefficient  $k_h$  variation with induction using four-points model (9) and CAL2 model – fully-processed material M43

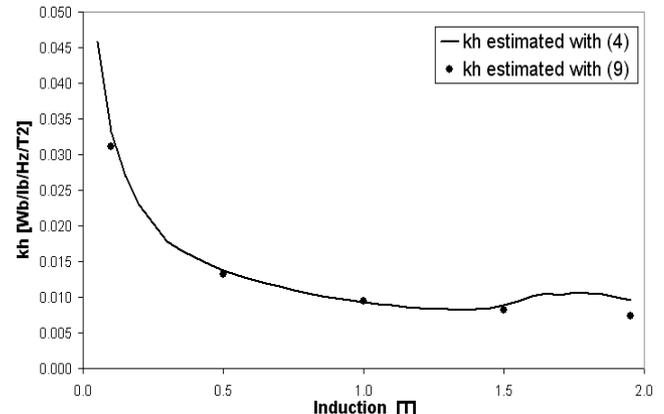


Fig. 12 Hysteresis loss coefficient  $k_h$  variation with induction using four-points model (9) and CAL2 model – semi-processed material SP

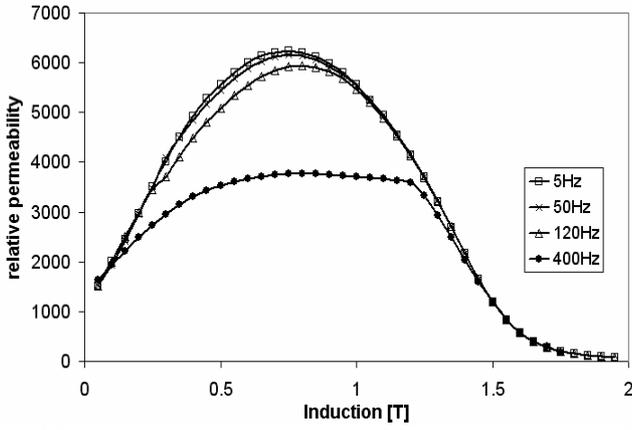


Fig. 13. Measured relative permeability curves vs peak induction for different frequency levels – fully-processed material M43

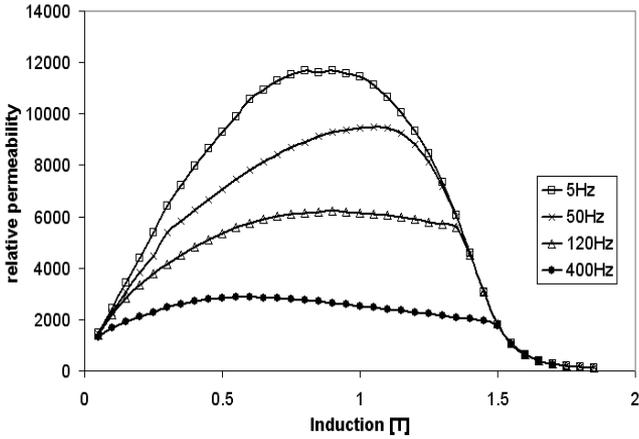


Fig. 14. Measured relative permeability curves vs peak induction for different frequency levels – semi-processed material SP

For the tested materials, the value calculated with (12) is:  $0.0000506[\text{Wb/lb}/\text{Hz}^2/\text{T}^2]$  for fully-processed material M43 and  $0.000114[\text{Wb/lb}/\text{Hz}^2/\text{T}^2]$  for semi-processed SP.

These values correspond practically to the average value of  $k_e$  from Figs. 5-6, low frequency  $< 400\text{Hz}$  polyfit curve, but it will be 50% higher than the average value obtained for the wide frequency polyfit curve. These differences suggest that the eddy-current loss coefficient must vary with frequency and induction.

*$k_e$  variation with frequency.*

In [9,12] was discussed the physical phenomenon of the field penetration into a ferromagnetic material and also the exact analytical solution was determined.

Thus, it is apparent that the finite depth penetration of the field  $z_0$  may be estimated with:

$$z_0 = \frac{\sqrt[4]{2n(3n+1)^2(n+1)}}{(n-1)\sqrt{2\pi f \mu_m \sigma}}, \quad n = 4, 5, \dots \quad (13)$$

where  $\mu_m$  is the magnetic permeability of the material. Associated with  $z_0$  is the skin-depth penetration, i.e. the distance where the field is decreased by a ratio of  $1/e$ :

$$\delta = \frac{1}{\sqrt{\pi f \mu_m \sigma}} \quad (14)$$

When the material thickness  $d > z_0$  we could modify (13) as a frequency dependent parameter:

$$k_e(f) = \frac{\pi^2 \sigma (z_0)^2}{6\rho_v} = K \frac{\pi}{f \mu_m} \quad (15)$$

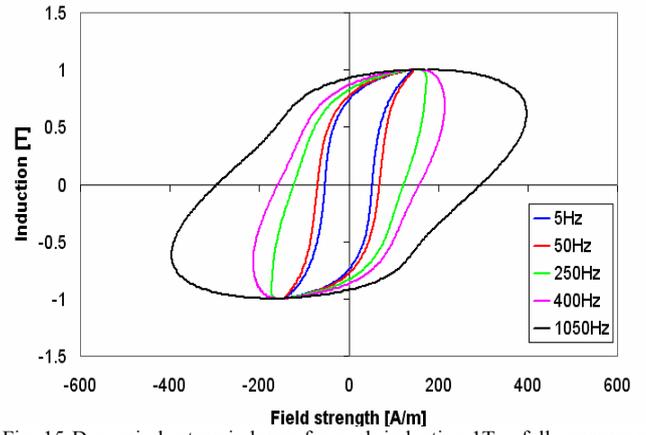


Fig. 15 Dynamic hysteresis loops for peak induction 1T – fully-processed material M43

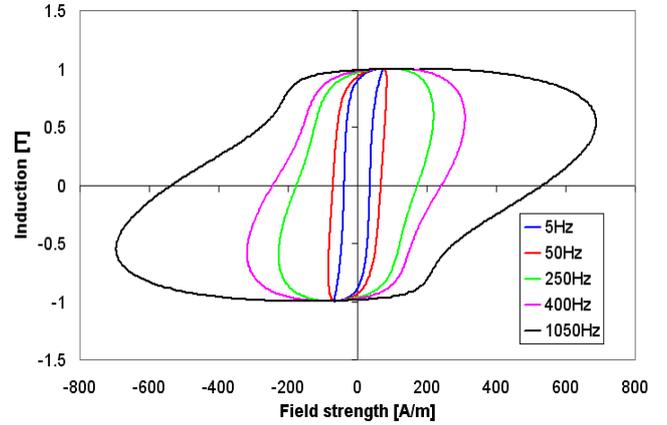


Fig. 16 Dynamic hysteresis loops for peak induction 1T – semi-processed material SP

where  $K$  is a material constant.

Relation (15) shows that if a wide range of frequencies is considered, the eddy-current loss coefficient will be inverse proportional to the product  $(f \mu_m)$ . In a simplified assessment we can consider that the material thickness  $d$  is reduced by the field penetration depth, or as it is widely defined the skin-effect. The new expression (15) will allow the eddy-currents to be ‘inductance limited’ a high frequency and also will allow the usage of the actual magnetic permeability in calculation of the rate-dependent core loss component.

In [5] and [11] the variation with frequency of the loss coefficient associated with the eddy-currents effect is taken into account using the expression:

$$k_e(f) = \frac{\pi^2 \sigma d^2}{2\lambda_i \rho_v} \cdot \frac{\sinh \lambda_i - \sin \lambda_i}{\cosh \lambda_i - \cos \lambda_i} \quad (16)$$

with:

$$\lambda_i = \frac{d}{\delta} \quad (17)$$

However, both expressions (15) and (16) lead to a small variation of the eddy-currents loss coefficient  $k_e$ . The variation of  $k_e(f)$  using (16) for both materials shows a maximum deviation of 0.035p.u. as compared to the classical loss coefficient (12).

*$k_e$  variation with induction.*

For low frequency range we have  $d < z_0$ , albeit the relation (11) gives a very close value to the average  $k_e$  as extracted from test data, the shape of the top curve from

Figs. 5-6 suggests that the eddy-currents loss coefficient varies with induction. By studying (12), we can identify as the only parameter that can vary with the induction – the electrical conductivity. It is known that this material property is variable with temperature, i.e. electrical conductivity decreases when the temperature increases and with the mechanical stress, i.e. a higher pressure determines a higher electrical conductivity [14,15].

However, the electrical conductivity variation with the induction in ferromagnetic lamination steel has not been studied. A proposed hypothesis for this phenomenon is as follows: as the material sample is subjected to increased input field strength  $H$  the instability of the magnetic domains is increasing and we can expect a major interaction between local adjacent domains and an increased Barkhausen effect.

How exactly the material conductivity will vary is a debatable phenomenon that is also linked to the material anisotropy.

### C. Minimum-effort modeling of the laminated steel losses

A practical approach for the determination of  $k_h$  and  $k_e$  according to the empirical model (5) and (6) is described as follows:

- measure the core losses corresponding to at least four induction values, i.e. 0.1T, 0.5T, 1T, 1.5T for a low frequency, i.e. 5Hz;
- determine experimental  $k_h$  with (9) for all four points;
- compute the terms  $k_{h1}$ ,  $k_{h2}$ ,  $k_{h3}$ ,  $k_{h4}$  from (6);
- measure the core losses corresponding to four induction values, i.e. 0.5T, 1T, 1.5T, 1.9T for the mean frequency of the interest range, e.g. 200Hz for a range up to 400Hz;
- determine experimental  $k_e$  for all four points with the expression:

$$k_e = \frac{1}{f^2 B_p^2} \left( w_{Fe} - f B_p \frac{\pi H_{irr}}{\rho_v} \right) \quad (18)$$

where  $H_{irr}$  is the irreversible field-strength measured at step (a) for low frequency, but similar induction values;

- compute the terms  $k_{e1}$ ,  $k_{e2}$ ,  $k_{e3}$ ,  $k_{e4}$  from (5);
- the loss coefficients are estimated with (5), (6) for any other induction level and may be easily further used in any numerical method of computing the core losses in electrical motors.
- the total core losses are computed with (2).

Figs. 17–18 show the comparison between the four-point values of  $k_e$  obtained with (18) vs. the predicted values of  $k_e$  using (3) and (4) for a large set of test data. Note the satisfactory agreement for all induction levels.

Figs. 19 – 22 show the variation of  $k_e$  when both the induction and frequency vary and the algorithm described above is employed. Note that as previously estimated the variation with frequency of the eddy-current loss coefficient is less significant, while an increased induction level will determine an increased value of  $k_e$ . We also observe that a fully-processed material will exhibit a lower degree of variation with induction and frequency as compared to the semi-processed material.

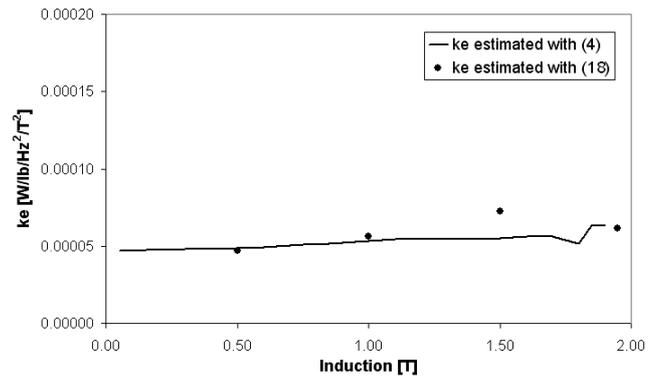


Fig. 17 Eddy-currents loss coefficient  $k_e$  variation with induction using at least four points model (18) and CAL2 model – fully-processed material M43

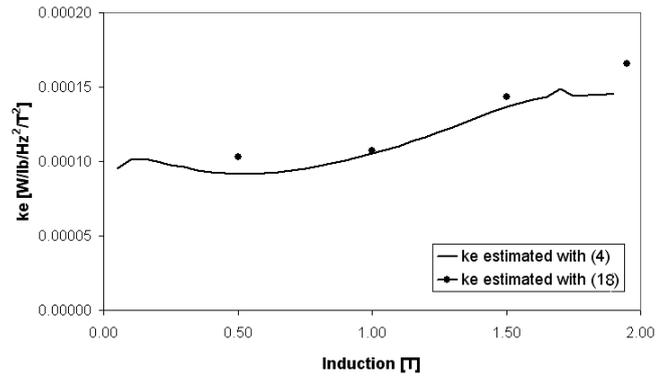


Fig. 18 Eddy-currents loss coefficient  $k_e$  variation with induction using at least four points model (18) and CAL2 model – semi-processed material SP

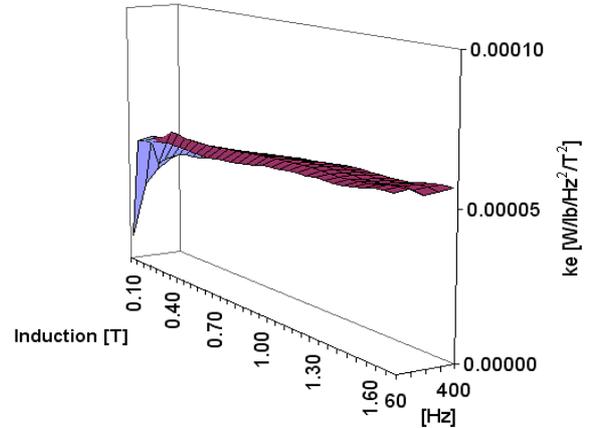


Fig. 19 Eddy-currents loss coefficient  $k_e$  variation with wide range induction and frequency as parameter – fully-processed material M43

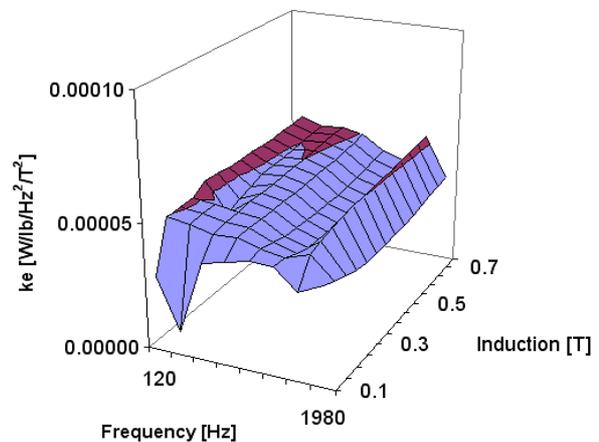


Fig. 20 Eddy-currents loss coefficient  $k_e$  variation with wide range frequency and induction as parameter – fully-processed material M43

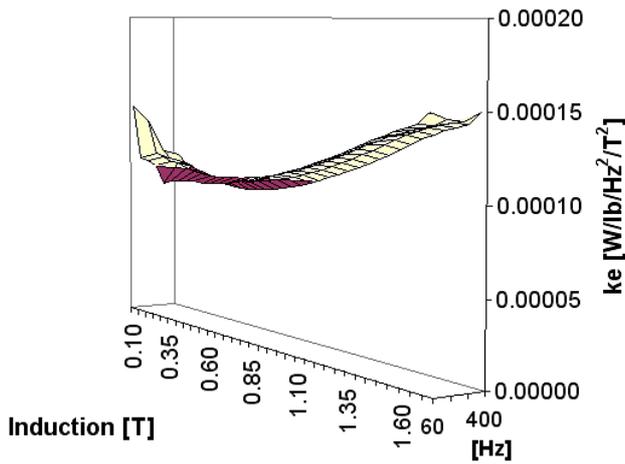


Fig. 21 Eddy-currents loss coefficient  $k_e$  variation with wide range induction and frequency as parameter – semi-processed material SP

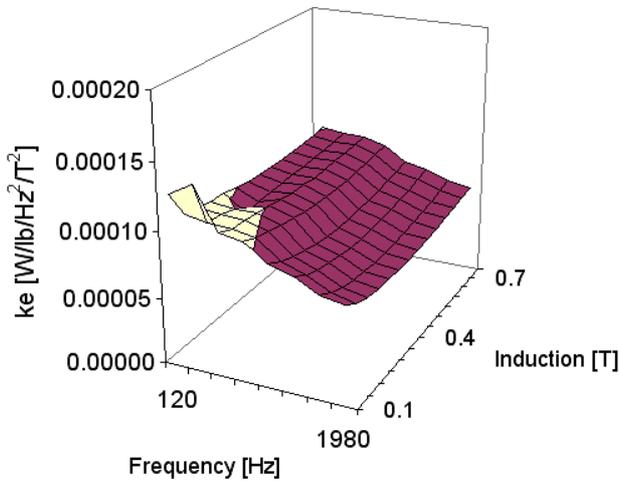


Fig. 22 Eddy-currents loss coefficient  $k_e$  variation with wide range frequency and induction as parameter – semi-processed material SP

#### IV. CONCLUSIONS

A physical explanation for the loss coefficients variation with frequency and induction, corresponding to the model proposed in [3] has been developed. The hysteresis loss coefficient ( $k_h$ ) accounts for the irreversible field strength  $H_{irr}$  and the non-linear permeability and this parameter are induction and frequency dependent, since  $H_{irr}$  depends on the induction and the permeability depends on frequency.

The classical eddy current coefficient ( $k_{e-classical}$ ) cannot model accurately the eddy-current losses for a wide range of frequencies and high induction level. The proposed coefficient ( $k_e$ ) reflects induction dependency assuming an empirical model of the electrical conductivity variation with the induction level. Also ( $k_e$ ) reflects frequency dependency due to skin effect at higher frequencies. Qualitatively, this could represent the dominant nature of the losses due to increased domain wall activity, which tend to form a strong shield against magnetization in the interior of the lamination. The separate effects of the induction and frequency on the eddy-currents loss coefficient are clearly identified. With only eight test points – four points for low frequency and four points for a frequency selected in the middle of the operation range – a complete and accurate

prediction of the core losses in any laminated steel material is achieved.

#### ACKNOWLEDGMENT

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#### APPENDIX

##### MAIN CHARACTERISTICS OF SAMPLE MATERIALS

Material type	Thickness	Permeability at 1.5T and 60Hz	Loss @ 1.5T and 60Hz	Density
	[in]	[-]	[W/lb]	[kg/m <sup>3</sup> ]
SP	0.022	3071	3.16	7850
M43	0.018	1387	1.88	7700

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