A Method for Determining IPM Motor Parameters from Simple Torque Test Data

Mircea Popescu  
Motor Design Limited  
Ellesmere, Shropshire, UK  
mircea.popescu@motor-design.com  

David Dorrell  
Faculty of Engineering and IT  
The University of Technology Sydney  
Sydney, Australia  
David.Dorrell@uts.edu.au

Abstract—This paper outlines a method for obtaining the parameters for the torque constant and the D-Q axes inductances, $L_d$ and $L_q$ for a brushless internal permanent magnet motor (IPM) by measurement of the static torque during testing. These are relatively simple tests to carry out. A review is made of the current theory. The new theory is full described and is based on well-known standard analysis. Simulations are carried out to test the theory against results obtained elsewhere. Good results are obtained. The advantage of this is that no detailed analysis is required and loading effects can be taken into account because $L_d$ and $L_q$ are not constant. The example machine used is a deeply embedded split-phase IPM with variable $L_d$ and $L_q$.

Keywords—permanent magnet motor, testing, torque

I. INTRODUCTION

The computation of a permanent magnet motor and its parameters has been the subject of many studies; [1] to [3] are examples of these. Many of these studies are quite in-depth in terms of assessing magnetization and losses [4][5]. Methods used for calculating the parameters include such techniques as finite element analysis [6] or simple on-line processes for estimations [7]. There are several texts on brushless permanent motors such as [8]. In terms of their operation they can be either DC controlled, or AC controlled; in the latter case standard synchronous machine theory such as that put forward in [9] can be utilized. To allow correct control the machine parameters are needed and often these can vary with different machines due to manufacturing variances, or simply the details of the machine are not known. There is therefore a requirement to be able to carry out a set of simple tests that can characterize the machine; and an ability to be able to do this without inverter control is advantageous. Tests were documented in [10] as well as several other papers but here we show a simple way in which two tests can be carried out with loads that are close to each other, and from these the machine parameters can be assessed. This allows the assessment of loading to be included. The theory is put forward and then an example is used to illustrate the method; this is a line start machine.

IPM motors are becoming increasingly common in many applications. These are the most popular choice for motor drive in automotive applications [11]-[13]. While design issues are addressed in many papers, simple testing procedures are still needed. In this paper we use an example of a line-start IPM which are used in applications such as washing machines. However, the method is still applicable for vehicle and renewable energy applications which are common applications for IPM machines are well as more standard servo motors.

II. ANALYSIS

A. Current Control

For a current controlled IPM the electromagnetic torque can be written as [8]:

$$ T_e = mp \left( \Psi_{Md} I_{ph} \cos(\gamma) + I_{ph}^2 \sin(2\gamma) \frac{L_q - L_d}{2} \right) $$  \hspace{1cm} (1)

where $m$ is the phase number, $p$ is the motor pole-pair number, $\Psi_{Md}$ is the magnet flux linkage on the d axis (rms), $I_{ph}$ is the phase current (rms) and $\gamma$ is the angle between the q axis and the current in electrical degrees. $L_d$ and $L_q$ are the d and q axis inductances. This can be rewritten

$$ T_e = A \cos(\gamma) + B \sin(2\gamma) $$  \hspace{1cm} (2)

If two static torque tests are carried out with the same current but different rotor angles (i.e., hold the rotor and inject d current to develop a torque) where the rotor angle is

$$ \theta_{ rotor} = \gamma / p $$  \hspace{1cm} (3)

so that

$$ T_1 = A \cos(\gamma_1) + B \sin(2\gamma_1) $$  \hspace{1cm} (4)

and

$$ T_2 = A \cos(\gamma_2) + B \sin(2\gamma_2) $$  \hspace{1cm} (5)

Ensure that the reference position is where the magnets align with the stator flux. From (4) and (5) we can solve $A$ and $B$ so that

$$ A = \frac{T_1 \sin(2\gamma_2) - T_2 \sin(2\gamma_1)}{\cos(\gamma_1) \sin(2\gamma_2) - \cos(\gamma_2) \sin(2\gamma_1)} $$  \hspace{1cm} (6)

$$ = m p \Psi_{Md} I_{ph} $$
and

\[
B = \frac{T_{12} \sin(\gamma_1) - T_{21} \sin(\gamma_2)}{\cos(\gamma_1) \sin(2\gamma_2) - \cos(\gamma_2) \sin(2\gamma_1)}
\]

(7)

From A we get

\[
\Psi_{Md} = \frac{A}{mpI_{ph}}
\]

(8)

and from B, we get

\[
L_q - L_d = \frac{2B}{mpI_{ph}}
\]

(9)

The inductances can be highly dependent on the current. It is possible to take several static tests at different current levels and phase angles and to examine the effect of loading on the inductances and indeed on the back-EMF constant. In the next section, voltage control is explored. This is not common but is increasing with the spread of line start motors and also brushless PM generators.

B. Voltage Control

Increasing voltage is being used to control the machine. This is relevant to line start motors that are becoming popular in smaller motors in domestic applications. For a voltage controlled motor (1) is adapted to

\[
T_e = \frac{mp}{\omega_s} \left( E_{q1} V_s \sin(\delta) + \frac{V_s^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin(2\delta) \right)
\]

(10)

This is derived from standard synchronous machine theory where \( \omega_s \) is the synchronous electrical frequency (rad/s), and \( \delta \) is the load angle between the induced fundamental back-EMF \( E_{q1} \) (usually called the excitation voltage) and the phase voltage \( V_s \). Note that we now deal with reactances rather than inductances because the supply frequency will be constant. The phase resistance is neglected in this analysis but can be simply included if necessary [9]. To maintain clarity in the analysis we can continue as stated that

\[
T_e = C \sin(\delta) + D \sin(2\delta)
\]

(11)

In this instance we can again carry out two load points but this time at the synchronous speed. In addition we need a reference no load point where the current is almost zero so that \( \delta = 0 \) and \( E_{q1} = V_s \). From the two load points:

\[
T_{11} = C \sin(\delta_1) + D \sin(2\delta_1)
\]

(12)

and

\[
T_{22} = C \sin(\delta_2) + D \sin(2\delta_2)
\]

(13)

We then obtain the equations:

\[
C = \frac{T_{11} \sin(2\delta_1) - T_{22} \sin(2\delta_2)}{\sin(\delta_1) \sin(2\delta_2) - \sin(\delta_2) \sin(2\delta_1)}
\]

(14)

and

\[
D = \frac{T_{21} \sin(\delta_1) - T_{11} \sin(\delta_2)}{\sin(\delta_1) \sin(2\delta_1) - \sin(\delta_2) \sin(2\delta_2)}
\]

(15)

From C we get

\[
X_q = \frac{mp E_{q1} V_s}{\omega_s C}
\]

(16)

but also, using D:

\[
\frac{1}{X_q} - \frac{1}{X_d} = \frac{2D\omega_s}{mpV_s^2}
\]

(17)

so that

\[
X_q = \left( \frac{2D\omega_s}{mpV_s^2} - \frac{\omega_s C}{mpE_{q1} V_s} \right)^{-1}
\]

(18)

Hence the machine parameters are now known.

C. Further Tests on Current Controlled Machine

For the current controlled machine then only the different inductances can be found. If a further rotation test is conducted, under voltage control then the D-axis inductance \( L_d \) can be obtained from a short circuit test by rotating the motor up to tested current. The back-EMF \( E_{q1} = \omega_s \Psi_{Md} \) and under short circuit

\[
X_q = \frac{E_{q1}}{I_q}
\]

(19)

Provided that the current phasor is aligned with the D axis, i.e., the resistance is negligible then

\[
L_d = \frac{\Psi_{Md}}{I_q} \frac{\Psi_{Md}}{I_{ph}}
\]

(20)

If the resistance is not negligible then the phase resistance can be measured and the current position measured using strobing and reference to open circuit voltage. From the phasor diagram in Fig. 1 then if can be seen that \( I_d = I_{ph} \sin(\gamma) \) and \( I_q = I_{ph} \cos(\gamma) \) so that we can equate the resistive components where

\[
I_q X_q = -I_d R_{ph}
\]

(21)

so that

\[
X_q = -\tan(\gamma) R_{ph}
\]

(22)

Hence the parameters can be found without use of a converter which leads to simple and quick bench testing.
III. TESTING OF METHOD

In this paper we will focus on one machine which is a two-phase unsymmetrical line start motor as reported by Walker [4]. A cross section is given in Fig. 2 and a specification in Table I. The machine was tested with a current of 2 A peak in the main phase. There are 970 turns in the main winding and 690 in the auxiliary. Due to the unsymmetrical magnetic structure, the impedances and currents are normalized to the main winding. Before examining the algorithm the parameters need to be assessed. In Fig. 3 the current – flux linkages loops are shown and this illustrates that the flux linkage term \( \Psi_{Md} \) is 1.125 V-s. However this includes the load and magnet flux. From open-circuit tests the magnet flux linkage was assessed to be 0.45 V-s. This values was calculated using finite element analysis and validated via measurements. The D-Q axis reactances were also calculated at 50 Hz and are shown in Fig. 4 [3]. It can be seen the strong variation of Q-axis reactance with current, while the D-axis reactance is less influenced by the current level.

A set of tests were carried out in which the current was maintained at 2 A peak and the torque angle (\( \gamma \)) varied with respect to the Q axis. The measured results are shown in Table II, in which using adjacent torque measurements to implement (6) and (7) above and the notional value of \( \gamma \) as the mean of the two measured values. Table III shows the values of the flux linkage \( \Psi_{Md} \) which are very close to the open circuit value and the differences \( X_q - X_d \) which are again close to the values calculated in Fig. 4. This validates the method.

![Fig. 1. Short circuit phasor diagram.](image1)

![Fig. 2. Line start motor cross section.](image2)

![Fig. 3. Current flux linkage loops.](image3)

<table>
<thead>
<tr>
<th>TABLE I. LINE START MOTOR SPECIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Rated voltage</td>
</tr>
<tr>
<td>Stack length</td>
</tr>
<tr>
<td>Lamination stacking factor</td>
</tr>
<tr>
<td>Shaft radius</td>
</tr>
<tr>
<td>Rotor outside radius</td>
</tr>
<tr>
<td>Air-gap length</td>
</tr>
<tr>
<td>Stator outside radius</td>
</tr>
<tr>
<td>Number of poles</td>
</tr>
<tr>
<td>Number of stator slots</td>
</tr>
<tr>
<td>Number of rotor bars</td>
</tr>
<tr>
<td>Rotor bridge thickness</td>
</tr>
<tr>
<td>Number of phases</td>
</tr>
<tr>
<td>Winding type</td>
</tr>
<tr>
<td>Magnet arc radius</td>
</tr>
<tr>
<td>Magnet type</td>
</tr>
<tr>
<td>Lamination material</td>
</tr>
</tbody>
</table>
### Table I. Measured Motor Loadings

<table>
<thead>
<tr>
<th>$I_{ph}$ (peak) [A]</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ [edeg]</td>
<td>-40</td>
<td>-20</td>
<td>-10</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>Torque [Nm]</td>
<td>-0.2722</td>
<td>0.6606</td>
<td>0.8401</td>
<td>0.9852</td>
<td>1.123</td>
<td>1.2301</td>
<td>1.2953</td>
</tr>
</tbody>
</table>

### Table II. Predicted Flux Constant and Reactance Difference

<table>
<thead>
<tr>
<th>$\gamma$ [edeg]</th>
<th>A</th>
<th>B</th>
<th>$\Psi_{Md}$ (peak) [A]</th>
<th>Xq-Xd [ohm]</th>
<th>Id (rms) [A]</th>
<th>Iq (rms) [A]</th>
<th>$E_q$ (rms) [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>1.098345</td>
<td>-0.76563</td>
<td>0.549172</td>
<td>-40.0885</td>
<td>-0.70709</td>
<td>1.224756</td>
<td>40.66395</td>
</tr>
<tr>
<td>-15</td>
<td>1.007826</td>
<td>-0.55393</td>
<td>0.503913</td>
<td>-29.0036</td>
<td>-0.36601</td>
<td>1.366028</td>
<td>37.31269</td>
</tr>
<tr>
<td>-5</td>
<td>0.9852</td>
<td>-0.5002</td>
<td>0.4926</td>
<td>-26.1904</td>
<td>-0.12325</td>
<td>1.408832</td>
<td>36.47499</td>
</tr>
<tr>
<td>5</td>
<td>0.9852</td>
<td>-0.5002</td>
<td>0.4926</td>
<td>-26.1904</td>
<td>0.12325</td>
<td>1.408832</td>
<td>36.47499</td>
</tr>
<tr>
<td>15</td>
<td>0.966316</td>
<td>-0.54707</td>
<td>0.483158</td>
<td>-28.6448</td>
<td>0.366015</td>
<td>1.366028</td>
<td>35.77584</td>
</tr>
<tr>
<td>30</td>
<td>0.87482</td>
<td>-0.80292</td>
<td>0.43741</td>
<td>-42.0409</td>
<td>0.707088</td>
<td>1.224756</td>
<td>32.38842</td>
</tr>
</tbody>
</table>

Fig. 4. $X_q$ against $I_q$ (with $I_d = 0$) and $X_d$ against $I_d$ (with $I_q = 0$).

### IV. Conclusions

There is an increasing use of brushless permanent magnet machines and to drive them correctly knowledge of the parameters is required. Fig. 5 shows a section of the Prius 2004 machine and this illustrates that the machine has high q-axis saliency. In later models the speed is increased and the magnet amount reduced so this becomes even more pronounced. Hence simple test methods are needed that are examined here. These will be useful to manufacturers who want to test their machines at either specific load points, or over a range of loads, to validate the magnetization and manufacture. Additionally it will be useful to users who have limited or no information about the machine and the tests will characterize the machine fully without the need of an inverter drive. In many generator applications diode bridges are used rather than inverters so these may not be available.

Fig. 5. Toyota Prius 2004 machine.

### References
