Abstract – A general model suitable for the vector control of the unsymmetrical two-phase induction machines is presented in the paper. Both the rotor flux oriented and the stator flux oriented control strategies are analysed and their suitability for low and high-speed operation is discussed. In the mathematical model the machine main and auxiliary winding are represented in the stationary reference frame and the effect of core losses and the main flux path saturation are included. A special technique is used to model different levels of saturation on the two axes of magnetisation. The model can be employed for the accurate simulation of both the steady-state and the transient operation of the unsymmetrical two-phase induction machines.

I. INTRODUCTION

The unsymmetrical two-phase induction machines are widely used in high volume commercial applications due to their relative low cost and high reliability. Conventionally, these machines are fed from a single-phase ac mains supply. To achieve variable speed operation a power electronics inverter can be used. Although in this case a configuration with a single phase input / three phase output inverter driving a three phase induction motor seems to be economically preferable, it is believed that for certain applications the true potential of the unsymmetrical two-phase induction motor drive is not yet exhausted.

The performance of a two-phase induction motor controlled by field oriented methods has been analysed in [1], based on a simplified linear non-referred model. It is known that iron losses and magnetic saturation are among the main causes of motor parameter variation, which – unless suitable compensatory measures are employed – can lead to significant performance deterioration in an electronically controlled drive. In this respect several modern control techniques have been proposed, but only for the three-phase induction motor drives [2]-[6]. A general theory for the unified treatment of core losses and saturation in the d-q models of electrical machines has been described in [7].

The main objective of this paper is to present an original method developed for de-tuning the effects of the iron losses and main flux path saturation in vector controlled unsymmetrical two phase induction motor drives, where the d-q motor parameters are different along the two axes. It will be shown that the method ensures satisfactory performance compensation and is suitable both for rotor flux and stator flux oriented control.

II. MATHEMATICAL MODEL

The proposed model is based on several assumptions, generally acceptable [8]:
1) only the fundamental space-harmonic component of the air-gap flux distribution is considered,
2) there is a unique distinct non-linear magnetisation curve along each of the two orthogonal axes of the machine (the magnetic-flux saturation effects are modelled and superposition is not used),
3) the core losses are modelled through a non-linear resistor associated with the total stator flux linkage,
4) the stray losses, the magnetic diffusion (i.e. deep-bar effect), the temperature effects and the rotational losses are neglected, but the model can be further developed to include them.

Since the stator windings of the unsymmetrical two-phase induction machine are not identical, the motor parameters are constant only in the stationary reference frame. In d-q coordinates, the machine model, represented in Fig. 1, is mathematically described through the following (the list of symbols is given in Appendix I):

- stator voltage equations
  \[ v_{qS} = R_S i_{qS} + L_{IM} p(i_{qS} - i_{qR}) + p \lambda_{qM} \]  
  \[ v_{dS} = R_S i_{dS} + L_{IM} p(i_{dS} - i_{dR}) + p \lambda_{dM} \]
- rotor voltage equations
  \[ 0 = R_R i_{qR} + L_{dR} p i_{qR} - \frac{1}{k} \omega i_{qR} + p \lambda_{qM} \]
  \[ 0 = k^2 R_R i_{dR} + k^2 L_{dR} p i_{dR} + k \omega, \lambda_{dR} + p \lambda_{dM} \]
- flux – linkage equations
  \[ \lambda_{qS} = L_{IM} (i_{qS} - i_{qM}) + \lambda_{qM} \]
  \[ \lambda_{dS} = L_{IM} (i_{dS} - i_{dR}) + \lambda_{dM} \]
  \[ \lambda_{qR} = L_{qR} i_{qR} + \lambda_{qM} \]
  \[ \lambda_{dR} = k^2 L_{dR} i_{dR} + \lambda_{dM} \]
  \[ \lambda_{qM} = (i_{qS} - i_{qR}) L_{qM} = L_{qM} (i_{qS} i_{qM}) \]
  \[ \lambda_{dM} = (i_{dS} - i_{dR}) L_{dM} = L_{dM} (i_{dS} i_{dM}) \]
- current equations
  \[ i_{qS} + i_{qR} = i_{qfe} + i_{qM} \]
  \[ i_{dS} + i_{dR} = i_{dfe} + i_{dM} \]
  \[ i_m = \sqrt{i_{qM}^2 + i_{dM}^2} \]

Because the machine is unsymmetrical, the time variation of the magnetisation flux-linkage in d-q axes is given by:
\[
\frac{d\lambda_d}{dt} = \left( \frac{d\lambda_{qm}}{di_m} i_d^2 m + \frac{d\lambda_{dm}}{di_m} i_m^2 \right) \frac{di_m}{dt} + \frac{d\lambda_{dm}}{di_m} \lambda_{dm} i_m \frac{di_m}{dt}
\]
\[
= \frac{dm}{dt} \rho = \frac{dm}{dt} \frac{\rho_m}{\rho_m}
\]
\[
\frac{d\lambda_q}{di_m} = \left( \frac{d\lambda_{qm}}{di_m} i_q^2 m + \frac{d\lambda_{qm}}{di_m} i_m^2 \right) \frac{di_m}{dt} + \frac{d\lambda_{dm}}{di_m} \lambda_{dm} i_m \frac{di_m}{dt}
\]
\[
= \frac{dm}{dt} \rho = \frac{dm}{dt} \frac{\rho_m}{\rho_m}
\]

By introducing the following notations:
\[
\frac{d\lambda_d}{di_m} = L_{dm} \frac{di_m}{dt}
\]
\[
\frac{d\lambda_q}{di_m} = L_{qm} \frac{di_m}{dt}
\]
\[
\lambda_{dm} = L_{dm}
\]
\[
\lambda_{qm} = L_{qm}
\]
the induced voltage in the magnetising branch becomes:
\[
v_d = \frac{d\lambda_d}{di_m} \frac{di_m}{dt} = L_{dm} \frac{di_m}{dt} \rho_m
\]
\[
v_q = \frac{d\lambda_q}{di_m} \frac{di_m}{dt} = L_{qm} \frac{di_m}{dt} \rho_m
\]

where the inductance terms are described by the relations:
\[
L_{dm} = L_{dm} + L_{dm} \cos \theta
\]
\[
L_{eq} = \frac{L_{dm} - L_{eq} \sin \theta}{2}
\]
\[
L_{qm} = \frac{L_{qm} + L_{qm} \cos \theta}{2}
\]
\[
\rho_m = \arctan \left( \frac{i_m}{i_m} \right)
\]

The electromagnetic torque equation and the mechanical movement equation are:
\[
T_e = \frac{P}{2} k \lambda_q i_d \left( i_q + i_q \right) - \frac{1}{k} L_{dm} \lambda_q \left( i_q + i_q - i_q \right) = \frac{P}{2} k \lambda_q i_d - \frac{1}{k} \lambda_q i_q
\]
\[
T_e - T_L = J_m \omega + B_m \omega
\]

The machine parameters can be computed, preferably by FEA, or measured through classical experimental methods. The value of the resistance used to model the iron loss is determined from a standard no-load test, at first with sinusoidal supply at rated frequency, and then with a PWM supply at rated frequency. The same test is used to measure discrete points on the magnetising curve of the machine. The methods for the measurement of the motor parameters are described in more detail in references [2]-[3].

Under transient operation, the system of equations (1)-(24) describes the performance of the two-phase induction motor.

Fig.1 The d-q axes equivalent circuit of the unsymmetrical two-phase induction machine including the iron loss and the main flux path saturation machine. The voltage matrix equation in state-variable form is [9]:
\[
v = A \bar{x} + B \bar{x}
\]
\[
\bar{x}^T = [i_{qS}, i_{dS}, i_{qR}, i_{dR}, i_{qS}, i_{dS}]
\]
\[
v^T = [v_{qS}, v_{dS}, 0, 0]
\]
where:
\[
A = \begin{bmatrix}
L_{qm} & L_{qS} & 0 & 0 & 0 & 0 \\
0 & L_{qS} & L_{qR} & 0 & 0 & 0 \\
L_{dS} & 0 & 0 & L_{dR} & 0 & 0 \\
0 & L_{dS} & 0 & L_{dR} & 0 & 0 \\
0 & 0 & L_{dR} & 0 & 0 & 0 \\
0 & 0 & 0 & L_{dR} & 0 & 0
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
R_{R} & 0 & 0 & 0 & 0 & 0 \\
0 & R_{R} & 0 & 0 & 0 & 0 \\
0 & 0 & R_{R} & 0 & 0 & 0 \\
0 & 0 & 0 & R_{R} & 0 & 0 \\
0 & 0 & 0 & 0 & R_{R} & 0 \\
0 & 0 & 0 & 0 & 0 & R_{R}
\end{bmatrix}
\]

The model described above was verified by comparing computed and measured results.

III. ROTOR FLUX VECTOR CONTROL OF AN UNSYMMETRICAL TWO-PHASE INDUCTION MACHINE

As the stator windings of the unsymmetrical two-phase induction motor are unbalanced, the machine parameters are different in between the d and q axis and the vector control principles have to be implemented in a special way. The waveform of the electromagnetic torque demonstrates that the system is unbalanced. Even if the orthogonal stator
currents $i_{qS}$ and $i_{dS}$ are of equal amplitude the torque contains an AC term as it can be seen from equation (23).

In order to eliminate the AC term from the torque expression some relations between the stator currents were proposed in [1]. However, these relations are valid only under linear assumptions (saturation and iron core losses are neglected). Furthermore, the model proposed in [1] was implemented using a non-referred equivalent circuit, which requires some complicated measurements of the rotor and stator magnetising and mutual inductance.

A new relation for computing the torque can be determined by substituting (7)-(10), in (23):

$$T_e = \left( \frac{p}{2} \frac{L_{qms}}{L_{dms}} \right) \left( \frac{1}{k} \lambda_{qR} i_{qS} - L_{qms} L_{dms} \frac{d}{dt} \lambda_{qR} + \frac{1}{k} L_{qms} i_{dS} \right)$$  \hspace{1cm} (29)

where:

$$i_{qS}' = i_{qS} - i_{dfe}$$
$$i_{dS}' = i_{dS} - i_{dfe}$$
$$L_{qR} = L_{IR} + L_{qms}$$
$$L_{dR} = k^2 L_{IR} + L_{dms}$$

As $L_{dms} = k^2 L_{qms}$, a torque expression without a steady-state oscillating term can be obtained by imposing the following conditions:

$$i_{dS}' = i_{dS}'_{dS1}$$  \hspace{1cm} (31)
$$i_{qS}' = k^2 \frac{L_{dms}}{L_{qms}} i_{dR}'_{dR1} = k^2 i_{qS1}$$  \hspace{1cm} (32)

By substituting the variables $i_{qS}'$ and $i_{dS}'$ into (29), the torque expression becomes:

$$T_e = \left( \frac{p}{2} \frac{k L_{dms}}{L_{qms}} \right) \left( \frac{1}{k} \lambda_{dR} i_{qS}'_{qS1} - \frac{1}{k} \lambda_{qR} i_{dS}'_{dS1} \right)$$  \hspace{1cm} (33)

Based on this new expression, which is equivalent to that of a symmetric machine, the vector control model can be implemented for the two-phase unsymmetrical machine.

From (3)-(4) and (7)-(10), the dynamic relation between the rotor flux and the stator currents is obtained as:

$$p \lambda_{qR} = - \frac{1}{\tau_{qR}} \lambda_{qR} + \frac{1}{k} \omega_e \lambda_{dR} + \frac{1}{\tau_{qR}} L_{qms} i_{qS1}$$  \hspace{1cm} (34)

$$p \lambda_{dR} = -k \omega_e \lambda_{qR} - \frac{1}{\tau_{dR}} \lambda_{dR} + \frac{1}{\tau_{dR}} L_{dms} i_{dS1}$$  \hspace{1cm} (35)

where:

$$\tau_{qR} = \frac{L_{qR}}{R_q}; \quad \tau_{dR} = \frac{L_{dR}}{k^2 R_q}$$

In an arbitrary reference frame, using (34)-(35) the rotor flux of a two-phase induction machine can now be derived as:

$$p \lambda_{qR}^* = - \frac{1}{\tau_{qR}} \lambda_{qR}^* + \frac{1}{k} (\omega_e - \omega_0) \lambda_{dR}^* + \frac{L_{dms}}{\tau_{qR}} i_{qS1}$$
$$p \lambda_{dR}^* = - \frac{1}{\tau_{dR}} \lambda_{dR}^* + k (\omega_e - \omega_0) \lambda_{qR}^* + \frac{L_{dms}}{\tau_{dR}} i_{dS1}$$  \hspace{1cm} (36-37)

where $\omega_0$ is the angular speed of the arbitrary reference frame.

**Rotor flux oriented control system.**

Based on the derived rotor flux model, the rotor field oriented control system can be adapted to a two-phase induction machine. The machine equations in the rotor flux reference frame can be obtained from (36) and (37) and then using the constraint condition for rotor flux field orientation control. The resulting equations are as follows:

$$\frac{\omega_e \lambda_{qR}}{\tau_{qR}} - \frac{L_{dms}}{\tau_{qR}} i_{qS1}$$
$$p \lambda_{dR} + \frac{1}{\tau_{dR}} \lambda_{qR} = \frac{L_{dms}}{\tau_{dR}} i_{dS1}$$  \hspace{1cm} (38-39)

where $\lambda_{dR}$ is the rotor flux amplitude. $\omega_e = \omega_0 - \omega_1$ and $\omega_0$ is the angular speed of the rotor flux as seen from the stator.

The torque expression in the rotor flux oriented control system becomes:

$$T_e = \left( \frac{p}{2} \right) \lambda_{dR} i_{qS1}$$  \hspace{1cm} (40)

Fig. 2 shows an indirect rotor flux oriented control system, with the references denoted by the * superscript.

The voltage source inverter and PWM module is fed by signals provide by current controllers (PI, fuzzy, or neural networks type). The input of these controllers is the difference between the estimated and measured values of the stator voltages, currents or networks type).

**IV. STATOR FLUX VECTOR CONTROL OF AN UNSYMETRICAL TWO-PHASE INDUCTION MACHINE**

The electromagnetic torque as a function of the stator flux-linkage and current is given by:

$$T_e = \left( \frac{p}{2} \right) \left[ \lambda_{qS} \lambda_{dS} \left( \frac{k}{L_{qms}} \frac{L_{dms}}{L_{qms}} \right) + \lambda_{qR} i_{qS} \left( \frac{k L_{dms}}{L_{qms}} \frac{L_{dms}}{L_{qms}} \right) + \lambda_{dR} i_{dS} \left( \frac{k L_{dms}}{L_{qms}} \frac{L_{dms}}{L_{qms}} \right) \right]$$  \hspace{1cm} (41)
The previous equation can be further simplified:

\[
T_c = \left( \frac{P}{2} \right) \left[ \frac{1}{k} \lambda_{qs} i_{qs}' - k \lambda_{qs} i_{qs}' + \left( k L_{qs} - \frac{1}{k} L_{qs} \right) i_{qs}' i_{qs}' \right] \quad (42)
\]

Usually, the stator main and auxiliary windings have an identical configuration, and differ only through the number of turns and the diameter of the copper wire. In this case, the ratio of the main and auxiliary leakage equals the turns ratio squared \((k^2)\). The conditions (31)-(32) for the stator currents become:

\[
i_{qs}' = k^2 i_{qs}' \quad ; \quad i_{ds}' = i_{ds}'
\]

Equation (42) can be re-written as:

\[
T_c \equiv \left( \frac{P}{2} \right) \left[ \lambda_{ds} i_{qs}' s - \lambda_{qs} i_{ds}' s \right] \quad (43)
\]

To express the dynamic relations as a function of the stator currents and fluxes, (5)-(8) can be substituted in (34)-(35), resulting in:

\[
p \lambda_{qs}' + \frac{1}{\tau_{ds}} \lambda_{qs} - k \omega_s \lambda_{qs} = k^2 \sigma_{qs} L_{qs} p i_{qs}' s - \quad (44)
\]

\[
- \frac{1}{k} \omega_s L_{qs} i_{qs}' s - \frac{k^2}{\tau_{ds}} L_{qs} i_{qs}' s \]

\[
p \lambda_{as}' + \frac{1}{\tau_{as}} \lambda_{as} + k \omega_s \lambda_{as} = \sigma_{as} L_{as} p i_{as}' s + \quad (45)
\]

\[+ k^3 \omega_s \sigma_{qs} L_{as} i_{qs}' s + \frac{1}{\tau_{as}} L_{as} i_{qs}' s \]

where \( \sigma_{qs} = 1 = \frac{L_{dqs}}{\omega_s} / \left( L_{dqs} L_{dqs} \right) \).

The dynamic equations for an arbitrary reference frame can be derived by substituting the \( \omega_s \) term with \( \omega_s - \omega_r \).

**Stator flux oriented control system.**

Based on the vector model given by (44)-(45) the field-oriented principles [10] can be applied to control the stator flux of the two-phase induction machine. The following equations are obtained:

\[
\lambda_{qs} = 0
\]

\[
\lambda_{as} = \lambda_s \quad (46)
\]

\[
i_{qs}' = \frac{2T_r}{P \lambda_s k}
\]

\[
p \lambda_s + \frac{1}{\tau_{ds}} \lambda_s = \sigma_{qs} L_{qs} p i_{qs}' s +
\]

\[+ k^3 \omega_s \sigma_{qs} L_{as} i_{qs}' s + \frac{1}{\tau_{as}} L_{as} i_{qs}' s \quad (49)
\]

**De-coupler for direct stator flux oriented machine.**

The \( d \)-axis component of the stator current for the stator flux oriented control system is not only dependent on the associated \( d \)-axis component of the stator flux, but also on the \( q \)-axis component of the stator current. The coupling between the torque producing current component \( i_{qs} \) and the flux producing current component \( i_{ds} \) is shown in (49). As a consequence, any change in one current component without changing the other accordingly will cause a transient in the stator flux. This disadvantage is overcome by using a de-coupler in order to compensate the coupling effect [10], [11].

The complete block diagram of the direct stator flux oriented system of two-phase induction machines is depicted in Fig. 3.

The description of the de-coupler is obtained as follows:

\[
i_{ds} = i_{ds} + F_j (p) \lambda_{as} = i_{ds} + \left( K_r \Delta \lambda_s + \frac{K_r \Delta \lambda_s}{p} \right)
\]

\[+ K_r \Delta \lambda_s + \frac{K_r \Delta \lambda_s}{p} \quad (50)
\]

![Fig. 2 Block diagram of indirect rotor flux oriented vector control system with two-phase induction machine](image-url)
where $F_f(p)$ is the transfer function of a PI type flux controller (soft computing techniques such as fuzzy or neural-network can also be employed), $K_p$ and $K_i$ are the coefficients of the PI type controller, and $\Delta \lambda_s = \lambda_{s}^{*} - \lambda_{es}$ is the difference between the reference and estimated value of stator flux.

In order to de-couple $i'_{dS1}$ from $i'_{qS1}$, the following has to be fulfilled:

$$3 \frac{d}{dt} q S d S d S d S d R s r d S d S q S 1 (') + = \omega \sigma \tau \omega \sigma + (51)$$

where $\omega_{sr} = \omega_e - \omega_r$ represents the slip angular frequency.

When properly tuned, the de-coupler described by the above relations generates correct values of stator estimated currents, so that the stator flux is not altered by any changes in the reference electromagnetic torque $T^*_{e}$, even during transients.

For a complete sensorless vector control of an unsymmetrical two-phase induction motor, the estimated stator flux is determined from the information provided by the terminal measured quantities (voltages and currents), as described in [2].

The electrical frequency is obtained through the relation:

$$\omega_e = \frac{\dot{\lambda}_{dS} - \dot{\lambda}_{dS} - p \dot{\lambda}_{dS}}{\dot{\lambda}_{dS}}$$

(53)

The effects of the magnetic saturation and core losses can be included in the machine model in such a manner that solely stator current governs the torque and flux production. It should be mentioned that in stator flux oriented control operation, the machine presents a steady-state pullout torque that limits the stable operating region.

V. EXPERIMENTAL AND COMPUTED RESULTS

An experimental validation of the proposed model was performed on a capacitor run two-phase induction motor, with the parameters given in Appendix II. The computed and the experimental torque–slip characteristic of the motor with both stator windings energised and no running capacitor are shown in Fig. 4. One should note that the rated and pull-out torque are less than expected for the case of the run capacitor connected, as the motor is designed to operate for such conditions. The direct connection of the two stator windings to the mains supply, was chosen in order to verify the model proposed in Fig. 1 without the capacitor influence.

To illustrate the comparative advantages of the proposed vector control strategy the same two-phase induction motor was simulated operating from mains supply with a running capacitor and operating in a vector control drive with no running capacitor, respectively. Due to space limitations, only a few simulation results are presented here. The torque against time and the torque against speed curves of the motor in line-start capacitor run operation are shown in Figs. 5-6, and the performance in vector control operation is displayed in Figs. 7 and 8.

Two important improvements are achieved through vector control operation: a significant reduction of the pulsating component of the motor torque and decayed speed oscillations around the steady-state operation point.

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Fig. 3 Block diagram of direct stator flux oriented vector control system with two-phase induction machine
VI. CONCLUSIONS

The equivalent circuit model proposed in the paper for the unsymmetrical two-phase machines includes both the effect of the magnetic saturation and the iron losses and therefore leads to a performance improvement of vector controlled machines. Further work will be undertaken in order to include into the model also the parameter variation with the temperature, the stray losses and the deep-bar effect.

The numerical results for the steady-state performance of an example capacitor-run two-phase induction motor agree satisfactorily with the experimental values. The simulated transient performance illustrates the advantages of the vector control drive in terms of reduced torque and speed ripple.

The vector control for this type of induction machine can be achieved either through a rotor or stator flux oriented strategy and the implementation can be done on the same DSP board because the same motor parameters are used in both versions. It is anticipated that the stator flux control is more suitable for low speed applications, while the rotor flux control system is applicable for high-speed applications.

APPENDIX I

List of principal symbols

- $\omega_e$: synchronous speed
- $\omega_r$: rotor speed
- $p$: differential operator
- $R_{M}$, $R_A$: main, auxiliary stator windings resistance
- $R_\psi$: rotor winding resistance
- $R_{dfe}$, $R_{qfe}$: equivalent iron-loss resistance (d and q axis)
- $L_{lM}$, $L_{lA}$: main, auxiliary stator leakage inductance
- $L_{dms,qms}$: magnetising inductance (d and q axis)
- $L_{qR}$: rotor leakage inductance
- $k$: turns ratio auxiliary/main windings
- $T_e$: electromagnetic torque
- $J_m$: rotor inertia
- $\lambda_{dS,qS, dR,qR}$: stator / rotor flux (d and q axis)
- $V_{dS,qS}$: stator voltages (d and q axis)
- $i_{dS,qS, dR,qR}$: stator / rotor current (d and q axis)
- $i_{dfe,qfe}$: iron-loss equivalent stator current
APPENDIX II

Two-phase induction motor data
Rated output power: 750 W
Rated frequency: 50 Hz
Rated speed: 1448 rpm
Rated voltage: 220 V
Number of poles: 4
Inertia: 0.00146 kgm²
Running capacitor: 10 µF/400 V
Stator main winding resistance: 5.35 Ω
Stator auxiliary winding resistance: 13.83 Ω
Rotor resistance: 3.95 Ω
Iron-loss resistance (in d axis): 1459 Ω
Iron-loss resistance (in q axis): 1287 Ω
Stator main winding leakage reactance: 12.35 Ω
Stator auxiliary winding reactance: 14.54 Ω
Rotor leakage reactance: 5.25 Ω
Mutual reactance (in d axis): 224.73 Ω
Mutual reactance (in q axis): 104.1 Ω

REFERENCES